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Approximate Solutions of a Generalized Hirota–Satsuma Coupled KdV and a Coupled mKdV Systems with Time Fractional Derivatives

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ABSTRACT

In the present study, a relatively new approach which is the combination of fractional complex transform and modified version of Adomian decomposition method also known as new iterative method is employed to get approximate solutions of time fractional generalized Hirota-Satsuma coupled KdV system and a coupled mKdV system. To find exact and approximate solutions of diverse forms of nonlinear fractional partial differential equations and systems present technique can be used as an alternative. Only a few terms are required to find an approximate solution which is compared with the exact solution to demonstrate the efficiency and accuracy of the method.

Keywords: Time fractional Hirota-Satsuma coupled KdV and coupled mkdV systems; Modified Riemann−Liouville definition; Fractional complex transforms; New iterative method. .

1. Introduction

The generalizations of ordinary and partial differential equations of integer orders to arbitrary order introduces new branch of mathematics known as fractional calculus. It plays an important part to model many phenomena in scientific fields such as physics, chemistry, mechanics, electrical networks, astronomy, diffusion, viscoelastic fluid, signal and image processing, reaction processes etc. (Hilfer (2000), Kilbas et al. (2000), Lakshmikantham et al. (2009) and Podlubny (1999)). Fractional-order models which are described by linear or nonlinear partial differential equations are of utmost importance and attracted many mathematicians and physicists to work on it.

These models are demonstrated using linear or nonlinear fractional differential equations and obtaining solutions of these equations is wide area of research and interest for many researcher. Because of complicated nature the exact analytical solutions of most of fractional differential equations does not exist, hence considerable focus is to get numerical and approximate solutions of these equations. The most commonly used methods to solve these equations are the Variational iteration method (He (1999b) and Yulita Molliq et al. (2009)), Adomian decomposition method (Adomian and Rach (1996) and Momani (2007)), Homotopy analysis method (Liao (2004), Song and Zhang (2009) and Gómez-Aguilar et al. (2016)), Laplace transform method (Sontakke and Shaikh (2015)), Laplace homotopy analysis method (Morales-Delgado et al. (2016)), Homotopy-perturbation method (He (1999a)), Homotopy perturbation transform method (Gómez-Aguilar et al. (2017)), Fractional complex transformation (Sontakke and Shaikh (2016c) and Sontakke and Shaikh (2016a)), Finite-difference method (Takale (2013) and Ren et al. (2013)), the (G'/G)-expansion method (Zheng (2012), Younis and Zafar (2013)), Feng's first integral method (Yépez-Martíneza et al. (2016)) etc. In recent years Daftardar-Gejji and Jafari (Daftardar-Gejji and Jafari (2006) and Sontakke and Shaikh (2016b)) have proposed new iterative method (NIM) which is one of the most effective and accurate algorithm to obtain solution in terms of rapidly convergent series of nonlinear partial differential equations.

Transformation method is an important mathematical approach and are applied in many solution procedure. With the help of this method, linear or nonlinear partial differential equations which have complicated solutions can be converted into well known differential equations. The converted equations are further solved by different methods present in advanced calculus. The fractional complex transform is introduced in (Li and He (2010), He and Li (2012) and He (2011)) consists of modified Riemann-Liouville derivatives (Jumarie (2009)). These transformation converts the fractional differential equation into ordinary differential equations in complex domain, which is further solved by any known method.

To explain wave propagation in the study of shallow water waves, systems of nonlinear partial differential equations is used. In 1981, R. Hirota and J. Satsuma introduced a coupled Korteweg-de Vries (KdV) equation known as the Hirota-Satsuma coupled KdV system. An interaction of two long waves with diverse dispersion relations is examined by these equations.

In recent times, many researchers have devoted considerable efforts by successfully implementing various methods to extract solitary wave solutions and other solutions of Hirota-Satsuma coupled KdV and a coupled mKdV systems (Fan (2001),Raslan (2004), Ganji et al. (2009) and Hu et al. (2013)). In (Wu et al. (1999)), Wu introduced a 4×4 matrix spectral problem with three potential and derived new hierarchy of nonlinear evolution equation which are a generalized Hirota−Satsuma coupled KdV (Korteweg-de Vries) equation and a coupled mKdV (modified Korteweg-de Vries) equation. In this work, we consider the time fractional generalized Hirota-Satsuma coupled KdV system and the coupled mKdV system presented by a system of partial differential equations to find numerical solution.

The time fractional generalized Hirota-Satsuma coupled KdV system is given as

$$\begin{cases} D_t^{\alpha} u = \frac{1}{2} u_{xxx} - 3uu_x + 3(vw)_x, \\ D_t^{\alpha} v = -v_{xxx} + 3uv_x, \qquad 0 < \alpha \leq 1, \\ D_t^{\alpha} w = -w_{xxx} + 3uw_x. \end{cases} \tag{1}$$

and a time fractional coupled mKdV system is

$$\begin{cases} D_t^{\alpha} u = \frac{1}{2} u_{xxx} - 3u^2 u_x + \frac{3}{2} v_{xx} + 3uv_x + 3u_x v - 3\lambda u_x, \\ D_t^{\alpha} v = -v_{xxx} - 3vv_x - 3u_x v_x + 3u^2 u_x + 3\lambda v_x, \qquad 0 < \alpha \leq 1. \end{cases} \tag{2}$$

The system of equations (1)and (2) becomes classical given in Wu et al. (1999) for $\alpha = 1$.

The paper is ordered as follows. In Section 2, we introduce some basic definitions. In Section 3, the fractional complex transform and the analysis of new iterative method is briefly presented. In Section 4, the approximate solutions, plots and absolute errors of the time fractional coupled KdV and mKdV systems are presented. Section 5 is the conclusions.

2. Basic definitions

In literature there are various definitions of fractional derivatives of which most frequently used are the Riemann-Liouville and Caputo derivatives of definitions.

Caputo's definition of derivative is given as:

$$D_t^{\alpha}(f(t)) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-x)^{n-\alpha-1} \frac{d^n}{dt^n} f(x)dx, \tag{3}$$

where $n-1 \leq \alpha < n, \quad n \in N \cup \{0\}$.

While Riemann−Liouville definition of derivative is given as:

$$D_t^{\alpha}(f(t)) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_0^t (t-x)^{n-\alpha-1} f(x)dx, \tag{4}$$

where $n-1 \leq \alpha < n, \quad n \in N \cup \{0\}$.

Every fractional derivative has its own benefits and drawbacks. For constant function the Riemann-Liouville derivative is non zero whereas, the Caputo derivative is zero. Caputo derivative operator can be applied on differentiable functions only but in case of Riemann-Liouville operator fractional derivatives of order less than one also exists. Recently, Jumarie (2006) proposed a simple alternate definition to the Riemann-Liouville derivative known as modified Riemann-Liouville derivative. It has advantages over previously defined operators and is denoted by the expression.

$$D_t^{\alpha}(f(t)) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dx^n} \int_0^t (t-x)^{n-\alpha-1} [f(x) - f(0)]dx, \tag{5}$$

it is defined over continuous but not necessarily differentiable functions.

3. Analysis of method

3.1 Fractional complex transform

This transformation is suggested by Li and He (2010) which is very easy solving procedure to convert the fractional derivative into integer order derivative and solved further by well known methods in advanced calculus.

We consider general fractional differential equation given as:

$$f(u, u_t^{(\alpha)}, u_x^{(\beta)}, u_y^{(\gamma)}, u_z^{(\lambda)}, u_t^{(2\alpha)}, u_x^{(2\beta)}, u_y^{(2\gamma)}, u_z^{(2\lambda)}, \cdots) = 0, \tag{6}$$

where $u_t^{(\alpha)} = \frac{\partial^\alpha u(x,y,z,t)}{\partial t^\alpha}$ represents modified Riemann−Liouville derivatives and $0 < \alpha \leq 1,\ 0 < \beta \leq 1,\ 0 < \gamma \leq 1,\ 0 < \lambda \leq 1.$

Introducing the following fractional complex transforms

$$T = \frac{mt^\alpha}{\Gamma(1+\alpha)},$$

$$X = \frac{nx^\beta}{\Gamma(1+\beta)},$$

$$Y = \frac{py^\gamma}{\Gamma(1+\gamma)},$$

$$Z = \frac{qz^\lambda}{\Gamma(1+\lambda)},$$

where m, n, p and$q$ are unknown constants. By adopting local fractional derivative and applying chain rule strongly Eq.(6) is converted into differential equation as below

$$f(u, u_T, u_X, u_Y, u_Z, u_{TT}, u_{XX}, u_{YY}, u_{ZZ}, \cdots) = 0. \tag{7}$$

Further this equation can be solved by the new iterative method which is described in next section.

3.2 New iterative method for a system of equations

In recent years, Daftardar-Gejji and Jafari (2006) introduced very simple and efficient method to solve linear and nonlinear differential equation known as New iterative method (NIM). This method is a modification of terms of Adomian decomposition method, so that it is easily implemented on computer with the help of symbolic computational packages like Mathematica.

Consider a system of equation

$$u_i(x,t) = f_i + L_i(u_1(x,t), u_2(x,t), \cdots, u_n(x,t)) + N_i(u_1(x,t), u_2(x,t), \cdots, u_n(x,t)), \quad (8)$$

for $i = 1, 2, \cdots, n$, where f_i is a known function, L_i and N_i are linear and non-linear operators respectively.

Let $u = (u_1(x,t), u_2(x,t), \cdots, u_n(x,t))$ be a solution of system of Eq. (8) where $u_i(x,t)$ is of the series form:

$$u_i(x,t) = \sum_{j=0}^{\infty} u_{i,j}(x,t) \qquad i = 1, 2, \cdots, n. \quad (9)$$

Since L is linear,

$$L_i\Big(\sum_{j=0}^{\infty} u_{i,j}(x,t)\Big) = \sum_{j=0}^{\infty} L_i(u_{i,j}(x,t)). \quad (10)$$

The nonlinear operator N_i is decomposed as

$$\begin{aligned} N_i(u) = N_i\left(\sum_{j=0}^{\infty} u_{i,j}(x,t)\right) &= N_i\left(\sum_{j=0}^{\infty} u_{1,j}(x,t), \cdots, \sum_{j=0}^{\infty} u_{n,j}(x,t)\right) \\ &= N_i(u_{1,0}(x,t), \cdots, u_{n,0}(x,t)) + \sum_{k=1}^{\infty}\Bigg\{N_i\Big(\sum_{j=0}^{k} u_{1,j}(x,t), \cdots, \sum_{j=0}^{k} u_{n,j}(x,t)\Big) \\ &\quad - N_i\Big(\sum_{j=0}^{k-1} u_{1,j}(x,t), \cdots, \sum_{j=0}^{k-1} u_{n,j}(x,t)\Big)\Bigg\}. \end{aligned} \quad (11)$$

In view of Eqs. (9), (10) and (11), the Eq.(8) is equivalent to

$$\begin{aligned}\sum_{j=0}^{\infty} u_{i,j}(x,t) = f_i + \sum_{j=0}^{\infty} L_i(u_{i,j}(x,t)) + N_i(u_{1,0}(x,t), \cdots, u_{n,0}(x,t))\\ + \sum_{k=1}^{\infty} \Bigg\{ N_i\Bigg(\sum_{j=0}^{k} u_{1,j}(x,t), \cdots, \sum_{j=0}^{k} u_{n,j}(x,t)\Bigg)\\ - N_i\Bigg(\sum_{j=0}^{k-1} u_{1,j}(x,t), \cdots, \sum_{j=0}^{k-1} u_{n,j}(x,t)\Bigg)\Bigg\}. \qquad (12)\end{aligned}$$

where $i = 1, 2, \cdots, n$. Further consider the recurrence relation as given below for $i = 1, 2, \cdots, n$

$$u_{i,0} = f_i,$$

$$u_{i,1} = L_i(u_{1,0}(x,t), \cdots, u_{n,0}(x,t)) + N_i(u_{1,0}(x,t), \cdots, u_{n,0}(x,t)),$$

$$\begin{aligned}u_{i,m+1} = \sum_{j=1}^{m} L_i(u_{i,j}(x,t)) + N_i \left(\sum_{j=0}^{m} u_{1,j}(x,t), \cdots, \sum_{j=0}^{m} u_{n,j}(x,t) \right) -\\ - N_i \left(\sum_{j=0}^{m-1} u_{1,j}(x,t), \cdots, \sum_{j=0}^{m-1} u_{n,j}(x,t) \right), \qquad (13)\end{aligned}$$

for $m = 1, 2, \cdots$. *The k-term approximate solution is given by*

$$u_i = u_{i,0} + u_{i,1} + u_{i,2} + \cdots + u_{i,k-1}. \qquad (14)$$

In Bhalekar and Daftardar-Gejji (2011), the detailed criteria of convergence of the series $\sum u_{i,j}$ is given.

4. Numerical application

To illustrate the applicability of method discussed in section 3, we consider the generalized time fractional Hirota-Satsuma coupled KdV equation and coupled mKdV equation.

4.1 Solution of the time fractional Hirota−Satsuma coupled KdV equation

Consider the system Eq.(1) with initial conditions as follows (Fan (2001))

$$\begin{cases} u(x,0) = \frac{\beta-2k^2}{3} + 2k^2 tanh^2(kx), \\ v(x,0) = -\frac{4k^2 c_0(\beta+k^2)}{3c_1^2} + \frac{4k^2(\beta+k^2)tanh(kx)}{3c_1}, \\ w(x,0) = c_0 + c_1 tanh(kx), \end{cases} \tag{15}$$

where k, $c_0, c_1 \neq 0$, and β are arbitrary constants.

The exact solution of Eq.(1) with initial condition (15) when $c = -\beta$ and $\alpha = 1$ is given as

$$\begin{cases} u(x,t) = \frac{\beta-2k^2}{3} + 2k^2 tanh^2(k(x-ct)), \\ v(x,t) = -\frac{4k^2 c_0(\beta+k^2)}{3c_1^2} + \frac{4k^2(\beta+k^2)tanh(k(x-ct))}{3c_1}, \\ w(x,t) = c_0 + c_1 tanh(k(x-ct)). \end{cases} \tag{16}$$

We consider the transformations

$$T = \frac{t^\alpha}{\Gamma(1+\alpha)}, \tag{17}$$

therefore

$$\frac{\partial^\alpha u}{\partial t^\alpha} = \frac{\partial u}{\partial T}, \qquad \frac{\partial^\alpha v}{\partial t^\alpha} = \frac{\partial v}{\partial T}, \qquad \frac{\partial^\alpha w}{\partial t^\alpha} = \frac{\partial w}{\partial T}.$$

Using this transformation (17) in Eq. (1) and operating I_T the corresponding equation becomes

$$u(x,T) = u(x,0)+L(u)+N(u),$$
$$v(x,T) = v(x,0)+L(v)+N(v),$$
$$w(x,T) = w(x,0)+L(w)+N(w),$$

where

$$L(u) = I_T(\frac{1}{2}u_{xxx}), \qquad N(u) = I_T(-3uu_x + 3(vw)_x),$$

$$L(v) = I_T(-v_{xxx}), \qquad N(v) = I_T(3uv_x),$$

and

$$L(w) = I_T(-w_{xxx}), \qquad N(w) = I_T(3uw_x).$$

Now taking series solution as $u(x,T) = \sum_{i=0}^{\infty} u_i(x,T)$, $v(x,T) = \sum_{i=0}^{\infty} v_i(x,T)$ and $w(x,T) = \sum_{i=0}^{\infty} w_i(x,T)$ then employing recursive relation Eq.(13) appropriately with initial condition (15) and substituting (17) we get an approximate solution as

$$\begin{cases} u_0 = \frac{2(-1+e^{2kx})^2k^2}{(1+e^{2kx})^2} + \frac{1}{3}(-2k^2+\beta) \\ \\ v_0 = -\frac{4k^2(k^2+\beta)c_0}{3c_1^2} + \frac{4(-1+e^{2kx})k^2(k^2+\beta)}{3(1+e^{2kx})c_1} \\ \\ w_0 = c_0 + \frac{(-1+e^{2kx})c_1}{1+e^{2kx}} \end{cases} \tag{18}$$

$$\begin{cases} u_1 = \frac{16e^{2kx}(-1+e^{2kx})k^3t^\alpha\beta}{(1+e^{2kx})^3\Gamma(1+\alpha)} \\ \\ v_1 = \frac{16e^{2kx}k^3t^\alpha\beta(k^2+\beta)}{3\Gamma(1+\alpha)(1+e^{2kx})^2c_1} \\ \\ w_1 = \frac{4e^{2kx}k\beta c_1t^\alpha}{(1+e^{2kx})^2\ \Gamma(1+\alpha)} \end{cases} \tag{19}$$

$$\begin{aligned} u_2 = {} & \frac{t^{2\alpha}k^6\beta[-32(e^{2kx}+e^{10kx})+832(e^{4kx}+e^{8kx})-2112e^{6kx}]}{(\Gamma(1+\alpha))^2(1+e^{2kx})^6} \\ & + \frac{t^{3\alpha}k^5\beta^3[256(e^{4kx}+e^{6kx}-e^{8kx}-e^{10kx})]}{3(\Gamma(1+\alpha))^3(1+e^{2kx})^7} + \frac{t^{2\alpha}k^4\beta^2[128(e^{6kx}+e^{8kx})]}{(\Gamma(1+\alpha))^2(1+e^{2kx})^7} \\ & + \frac{t^{2\alpha}k^4\beta^2[16(e^{2kx}+e^{4kx}+e^{10kx}-e^{12kx})]+t^{2\alpha}k^6\beta[32(e^{2kx}+e^{12kx})]}{(\Gamma(1+\alpha))^2(1+e^{2kx})^7} \\ & + \frac{t^{3\alpha}k^7\beta^2[1280(-e^{4kx}+e^{10kx})]+7936(e^{6kx}-e^{8kx})}{3(\Gamma(1+\alpha))^3(1+e^{2kx})^7} \\ & + \frac{t^{2\alpha}k^6\beta[-800(e^{4kx}+e^{10kx})+1280(e^{6kx}+e^{8kx})]}{(\Gamma(1+\alpha))^2(1+e^{2kx})^7}, \end{aligned}$$

$$v_2 = \frac{t^{2\alpha}[(e^{2kx} - e^{10kx})(64k^8\beta + 80k^6\beta^2 + 16k^4\beta^3)]}{3(\Gamma(1+\alpha))^2(1+e^{2kx})^6\ c_1} + \\ + \frac{t^{2\alpha}[k^6\beta^2 + k^8\beta][64(e^{8kx} - e^{2kx}) + 704(e^{4kx} - e^{6kx})]}{3(\Gamma(1+\alpha))^2(1+e^{2kx})^5\ c_1} \\ + \frac{t^{3\alpha}[(k^9\beta^2 + k^7\beta^3)(-512(e^{4kx} + e^{8kx}) + 1024e^{6kx})]}{3(\Gamma(1+\alpha))^3(1+e^{2kx})^6 c_1} \\ + \frac{t^{2\alpha}[(e^{8kx} - e^{4kx})(640k^8\beta + 608k^6\beta^2 - 32k^4\beta^3)]}{3(\Gamma(1+\alpha))^2(1+e^{2kx})^6\ c_1} \\ + \frac{t^{3\alpha}[(k^9\beta^2 + k^7\beta^3)(-512(e^{4kx} + e^{8kx}))]}{3(\Gamma(1+\alpha))^3(1+e^{2kx})^6 c_1},$$

$$w_2 = \frac{t^{2\alpha}k^4\beta c_1[16(e^{2kx} - e^{10kx}) + 160(e^{8kx} - e^{4kx})]}{(\Gamma(1+\alpha))^2(1+e^{2kx})^6} \\ + \frac{t^{2\alpha}k^2\beta^2 c_1[4(e^{2kx} - e^{10kx}) + 8(e^{4kx} - e^{8kx})]}{(\Gamma(1+\alpha))^2(1+e^{2kx})^6} \\ + \frac{t^{2\alpha}k^4\beta c_1[16(e^{8kx} - e^{2kx}) + 176(e^{4kx} - e^{6kx})]}{(\Gamma(1+\alpha))^2(1+e^{2kx})^5} \\ + \frac{t^{3\alpha}k^5\beta^2 c_1[-128(e^{4kx} + e^{8kx}) + 256e^{6kx}]}{(\Gamma(1+\alpha))^3(1+e^{2kx})^6}. \tag{20}$$

Successively applying the algorithm given in Eq. (13), initial few terms of $u(x,t), v(x,t)$and$w(x,t)$ can be obtained from software package Mathematica. The approximate solution in series form is given as

$$\begin{cases} u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t), \\ v(x,t) = v_0(x,t) + v_1(x,t) + v_2(x,t), \\ w(x,t) = w_0(x,t) + w_1(x,t) + w_2(x,t). \end{cases} \tag{21}$$

Figures 1, 3 and 5 show the surfaces of approximate solution of Eq. (1) for u(x,t) which is of bell shaped but kink-type for $v(x,t)$and$w(x,t)$ when $\alpha = 1$, $\beta = 1.5$, $k = 0.1$, $c_0 = 1.5$ and $c_1 = 0.1$ respectively also Figures 2, 4 and 6 shows the plot of $u(x,t), v(x,t)$and$w(x,t)$ of Eq.(1) for $k = 0.1$, $\alpha = 1.5$, $c_0 = 1.5$ and $c_1 = 0.1$, $-50 \leq x \leq 50$ and $t = 0.5$ respectively. It seems that all the curves of approximate solution are exactly similar with the curves of exact solutions (Fan (2001)).

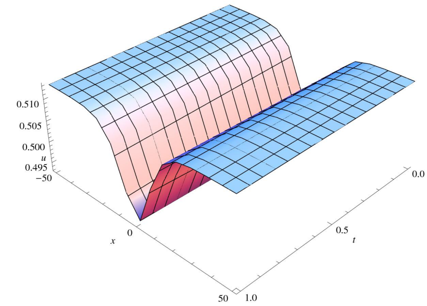

Figure 1: Approx. sol. of u(x,t) of (1) for $\alpha = 1,\ k = 0.1,\ \beta = 1.5$

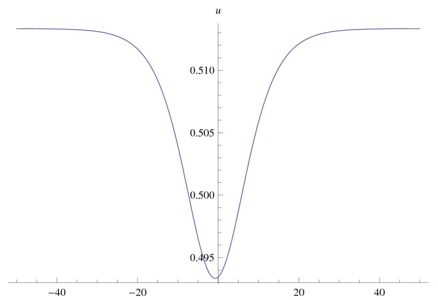

Figure 2: Plot of u(x,t) of (1) for $\alpha = 1,\ t = 0.5$

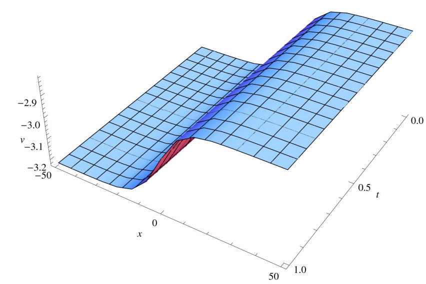

Figure 3: Approx. sol. of v(x,t) of (1) for $\alpha = 1,\ k = 0.1,\ \beta = 1.5$

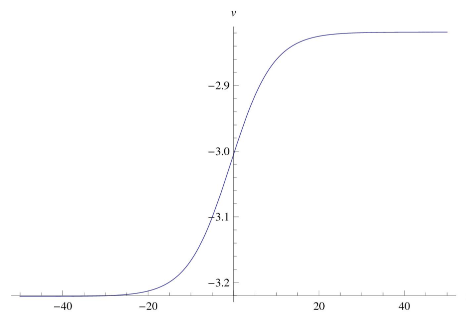

Figure 4: Plot of v(x,t) of (1) for $\alpha = 1,\ t = 0.5$

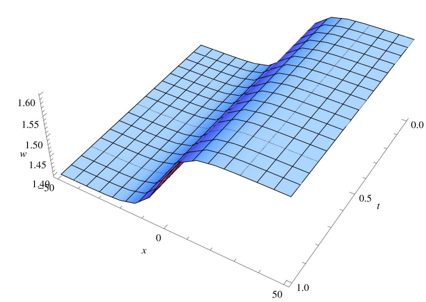

Figure 5: Approx. sol. of w(x,t) of (1) for $\alpha = 1,\ k = 0.1,\ \beta = 1.5$

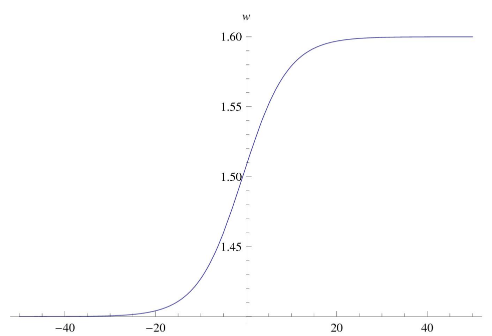

Figure 6: Plot of w(x,t) of (1) for $\alpha = 1,\ t = 0.5$

In Table 1, 2 and 3 the above series solutions are tested by evaluating absolute errors between approximate solution of coupled KdV system Eq. (1) with exact solutions (Fan (2001)) given by (16). We substitute $\alpha = 1$, $\beta = 1.5$, $k = 0.1$, $c_0 = 1.5$ and $c_1 = 0.1$ in approximate solution and take only three terms of all series. Also numerical values of solution are evaluated for $\alpha = 0.5$, and 0.75. From these results it has been observed that the obtained approximate series solutions are in good agreement with the exact solutions.

Table 1: The comparison of numerical solution of u(x,t) of (1) obtained by NIM with HPM (Ganji et al. (2009)), also absolute errors for difference between exact and numerical solution for $\beta = 1.5$, $k = 0.1$, $c_0 = 1.5$, $c_1 = 0.1$ and $\alpha = 0.5, 0.75, 1$

		$\alpha = 0.5$		$\alpha = 0.75$		$\alpha = 1$		Absolute error
t	x	u_{HPM}	u_{NIM}	u_{HPM}	u_{NIM}	u_{HPM}	u_{NIM}	$\|u_{exact} - u_{appx}\|$
0.2	0	0.49335133	0.49344792	0.49335133	0.49338098	0.49335133	0.49335133	1.07975×10^{-8}
	0.25	0.49341365	0.49353517	0.49340604	0.49344197	0.49339376	0.49339372	9.74133×10^{-9}
	0.5	0.49350055	0.49364640	0.49348538	0.49352741	0.49346087	0.49346079	8.52453×10^{-9}
	0.75	0.49361162	0.49378108	0.49358894	0.49363687	0.49355233	0.49355223	7.09823×10^{-9}
	1	0.49374629	0.49393853	0.49371623	0.49376982	0.49366771	0.49366757	5.42089×10^{-9}
0.4	0	0.49340533	0.49356252	0.49340533	0.49346811	0.49340533	0.49340533	1.72448×10^{-7}
	0.25	0.49350903	0.49367972	0.49349547	0.49356153	0.49347759	0.49347730	1.62910×10^{-7}
	0.5	0.49363683	0.49382019	0.49360978	0.49367881	0.49357413	0.49357355	1.51173×10^{-7}
	0.75	0.49378812	0.49398326	0.49374772	0.49381937	0.49369446	0.49369359	1.36904×10^{-7}
	1	0.49396215	0.49416812	0.49390859	0.49398253	0.49383799	0.49383684	1.19824×10^{-7}
0.6	0	0.49349533	0.49367711	0.49349533	0.49358093	0.49349533	0.49349533	8.70802×10^{-7}
	0.25	0.49363487	0.49381666	0.49361742	0.49370209	0.49359735	0.49359635	8.33414×10^{-7}
	0.5	0.49379790	0.49397880	0.49376308	0.49384642	0.49372303	0.49372105	7.85653×10^{-7}
	0.75	0.49398361	0.49416276	0.49393159	0.49401320	0.49387178	0.49386883	7.26548×10^{-7}
	1	0.49419110	0.49436767	0.49412215	0.49426165	0.49404286	0.49403896	6.55344×10^{-7}

Table 2: The comparison of numerical solution of v(x,t) of (1) obtained by NIM with HPM (Ganji et al. (2009)), also absolute errors for difference between exact and numerical solution for $\beta = 1.5$, $k = 0.1$, $c_0 = 1.5$, $c_1 = 0.1$ and $\alpha = 0.5, 0.75, 1$

		$\alpha = 0.5$		$\alpha = 0.75$		$\alpha = 1$		Absolute error
t	x	v_{HPM}	v_{NIM}	v_{HPM}	v_{NIM}	v_{HPM}	v_{NIM}	$\|v_{exact} - v_{appr}\|$
0.2	0	-3.0099519	-3.0047602	-3.0114850	-3.0101727	-3.0139600	-3.0139600	1.81135×10^{-6}
	0.25	-3.0049304	-2.9997663	-3.0064626	-3.0051585	-3.0089360	-3.0089360	1.80393×10^{-6}
	0.5	-2.9999278	-2.9947975	-3.0014570	-3.0001628	-3.0039258	-3.0039258	1.78719×10^{-6}
	0.75	-2.9949500	-2.9898598	-2.9964744	-2.9951917	-2.9989356	-2.9989356	1.76130×10^{-6}
	1	-2.9900032	-2.9849591	-2.9915210	-2.9902511	-2.9939714	-2.9939714	1.72655×10^{-6}
0.4	0	-3.0015872	-2.9984478	-3.0043191	-3.0034725	-3.0079200	-3.0079200	1.44752×10^{-5}
	0.25	-2.9965846	-2.9934865	-2.9993147	-2.9984844	-3.0029134	-3.0029133	1.43943×10^{-5}
	0.5	-2.9916110	-2.9885582	-2.9943361	-2.9935231	-2.9979279	-2.9979279	1.42392×10^{-5}
	0.75	-2.9866726	-2.9836686	-2.9893891	-2.9885944	-2.9929699	-2.9929699	1.40116×10^{-5}
	1	-2.9817752	-2.9788231	-2.9844799	-2.9837041	-2.9880450	-2.9880450	1.37138×10^{-5}
0.6	0	-2.9943184	-2.9936040	-2.9978355	-2.9975986	-3.0018800	-3.0018800	4.87660×10^{-5}
	0.25	-2.9893429	-2.9886746	-2.9928587	-2.9926426	-2.9968998	-2.9968997	4.84210×10^{-5}
	0.5	-2.9844055	-2.9837841	-2.9879138	-2.9877205	-2.9919482	-2.9919482	4.78272×10^{-5}
	0.75	-2.9795119	-2.9789379	-2.9830094	-2.9828380	-2.9870312	-2.9870312	4.69909×10^{-5}
	1	-2.9746679	-2.9741414	-2.9781501	-2.9780007	-2.9821544	-2.9821545	4.59211×10^{-5}

Table 3: The comparison of numerical solution of w(x,t) of Eq.(1) obtained by NIM with HPM (Ganji et al. (2009)), also absolute errors for difference between numerical solution for $\beta = 1.5,\ k = 0.1,\ c_0 = 1.5,\ c_1 = 0.1$ and $\alpha = 0.5,\ 0.75,\ 1$ and exact solution

		$\alpha = 0.5$		$\alpha = 0.75$		$\alpha = 1$		Absolute error
t	x	w_{HPM}	w_{NIM}	w_{HPM}	w_{NIM}	w_{HPM}	w_{NIM}	$\lvert w_{exact} - w_{appx}\rvert$
0.2	0	1.50499074	1.50756939	1.50422929	1.50488111	1.50300000	1.50300000	8.99676×10^{-7}
	0.25	1.50748486	1.51004983	1.50672388	1.50737159	1.50549536	1.50549536	8.95993×10^{-7}
	0.5	1.50996964	1.51251778	1.50921009	1.50985289	1.50798386	1.50798386	8.87678×10^{-7}
	0.75	1.51244205	1.51497028	1.51168486	1.51232198	1.51046246	1.51046246	8.74820×10^{-7}
	1	1.51489908	1.51740444	1.51414519	1.51477591	1.51292811	1.51292812	8.57559×10^{-7}
0.4	0	1.50914540	1.51070474	1.50778852	1.50820902	1.50600000	1.50600000	7.18965×10^{-6}
	0.25	1.51163017	1.51316891	1.51027414	1.51068653	1.50848674	1.50848674	7.14948×10^{-6}
	0.5	1.51410047	1.51561674	1.51274697	1.51315078	1.51096292	1.51096292	7.07245×10^{-6}
	0.75	1.51655332	1.51804539	1.51520404	1.51559880	1.51342555	1.51342554	6.95939×10^{-6}
	1	1.51898582	1.52045207	1.51764242	1.51802775	1.51587167	1.51587166	6.81151×10^{-6}
0.6	0	1.51275576	1.51311058	1.51100884	1.51112653	1.50900000	1.50900000	2.42215×10^{-5}
	0.25	1.51522704	1.51555893	1.51348121	1.51358813	1.51147362	1.51147362	2.40502×10^{-5}
	0.5	1.51767940	1.51798800	1.51593684	1.51603288	1.51393300	1.51393300	2.37552×10^{-5}
	0.75	1.52010996	1.52039506	1.51837282	1.51845795	1.51637524	1.51637522	2.33399×10^{-5}
	1	1.52251593	1.52277741	1.52078635	1.52086056	1.51879747	1.51879743	2.28085×10^{-5}

4.2 Solution of the time fractional coupled mKdV equation

We consider the system of Eq. (2) with initial conditions as follows (Fan (2001)):

$$\begin{cases} u(x,0) = \frac{k(-1+e^{2kx})}{(1+e^{2kx})}, \\ v(x,0) = \frac{4k^2+\lambda}{2} - \frac{2k^2(-1+e^{2kx})^2}{(1+e^{2kx})^2}. \end{cases} \tag{22}$$

The exact solution of Equation (2) with initial condition (22) is (Fan (2001)):

$$\begin{cases} u(x,t) = \frac{(-1+e^{2k(x-tk^2-\frac{3t\lambda}{2})})k}{(1+e^{2k(x-tk^2-\frac{3t\lambda}{2})})}, \\ v(x,t) = \frac{4k^2+\lambda}{2} - \frac{2k^2(-1+e^{2k(x-tk^2-\frac{3t\lambda}{2})})^2}{(1+e^{2k(x-tk^2-\frac{3t\lambda}{2})})^2}. \end{cases} \tag{23}$$

We consider the transformations

$$T = \frac{t^\alpha}{\Gamma(1+\alpha)} \tag{24}$$

Therefore

$$\frac{\partial^\alpha u}{\partial t^\alpha} = \frac{\partial u}{\partial T}, \qquad \frac{\partial^\alpha v}{\partial t^\alpha} = \frac{\partial v}{\partial T}.$$

Using this transformation (24) in Eq. (2) and operating I_T the corresponding equation becomes

$$u(x,T) = u(x,0) + L(u) + N(u),$$

$v(x,T) = v(x,0) + L(v) + N(v)$,

where

$$L(u) = I_T(\frac{1}{2}u_{xxx} + \frac{3}{2}v_{xx} - 3\lambda u_x), \qquad N(u) = I_T(-3u^2u_x + 3u_xv + 3uv_x)$$

$$L(v) = I_T(-v_{xxx} + 3\lambda v_x), \qquad N(v) = I_T(3u^2v_x - 3vv_x - 3u_xv_x)$$

Now taking series solution as $u(x,T) = \sum_{i=0}^{\infty} u_i(x,T)$ and $v(x,T) = \sum_{i=0}^{\infty} v_i(x,T)$ then employing recursive relation Eq.(13) appropriately with initial condition (22) and substituting (24) we get approximate solution as

$$\begin{cases} u_0 = \frac{k(-1+e^{2kx})}{(1+e^{2kx})}, \\ v_0 = \frac{4k^2+\lambda}{2} - \frac{2k^2(-1+e^{2kx})^2}{(1+e^{2kx})^2}. \end{cases} \tag{25}$$

$$\begin{cases} u_1 = -\frac{2e^{2kx}k^2t(2k^2+3\lambda)}{(1+e^{2kx})^2}, \\ v_1 = \frac{8e^{2kx}k^3t(-1+e^{2kx})(2k^2-3\lambda)}{(1+e^{2kx})^3}. \end{cases} \tag{26}$$

$$\begin{aligned} u_2 = {} & \frac{t^{2\alpha}[(e^{2kx} - e^{12kx})(60k^7 - 60k^5\lambda - 9k^3\lambda^2) + (e^{10kx} - e^{4kx})492k^7]}{(\Gamma(1+\alpha))^2(1+e^{2kx})^7} \\ & + \frac{t^{2\alpha}[(e^{10kx} - e^{4kx})(36k^5\lambda + 27k^3\lambda^2) + (e^{8kx} - e^{6kx})(552k^7 - 24k^5\lambda)]}{(\Gamma(1+\alpha))^2(1+e^{2kx})^7} \\ & + \frac{t^{2\alpha}[(e^{8kx} - e^{6kx})(18k^3\lambda^2)]}{(\Gamma(1+\alpha))^2(1+e^{2kx})^7} + \frac{t^{2\alpha}[(e^{4kx} - e^{6kx})(-648k^5\lambda + 18k^3\lambda^2)]}{(\Gamma(1+\alpha))^2(1+e^{2kx})^5} \\ & + \frac{t^{2\alpha}[(e^{2kx} - e^{8kx})(-56k^7 + 72k^5\lambda + 18k^3\lambda^2) + (e^{4kx} - e^{6kx})616k^7]}{(\Gamma(1+\alpha))^2(1+e^{2kx})^5} \\ & + \frac{t^{3\alpha}[(e^{6kx} + e^{8kx})(-620k^{10} - 384k^8\lambda + 864k^3\lambda^2) + (e^{4kx})320k^{10}]}{(\Gamma(1+\alpha))^3(1+e^{2kx})^7} \\ & + \frac{t^{4\alpha}[(e^{6kx} - e^{8kx})(648k^9\lambda^2 + 324k^7\lambda^3 + 96k^3 + 432k^{11}\lambda)]}{(\Gamma(1+\alpha))^4(1+e^{2kx})^7} \\ & + \frac{t^{3\alpha}[(e^{4kx} + e^{10kx})(192k^8\lambda - 432k^6\lambda^2)]}{(\Gamma(1+\alpha))^3(1+e^{2kx})^7}, \end{aligned}$$

$$
\begin{aligned}
v_2 = & \frac{t^{2\alpha}[(e^{6kx}+e^{10kx})(1440k^6\lambda+324k^4\lambda^2)]}{(\Gamma(1+\alpha))^2(1+e^{2kx})^8} + \frac{t^{4\alpha}[e^{8kx}2304k^{12}\lambda]}{(\Gamma(1+\alpha))^4(1+e^{2kx})^8} \\
& + \frac{t^{2\alpha}[(e^{2kx}+e^{10kx})(64k^8-144k^6\lambda+72k^4\lambda^2)+(e^{4kx}+e^{8kx})-1664k^8]}{(\Gamma(1+\alpha))^2(1+e^{2kx})^6} \\
& + \frac{t^{4\alpha}[(e^{6kx}+e^{10kx})(-384k^{14}-576k^{12}\lambda+864k^{10}\lambda^2+1296k^8\lambda^3)]}{(\Gamma(1+\alpha))^4(1+e^{2kx})^8} \\
& + \frac{t^{2\alpha}[(e^{4kx}+e^{12kx})(536k^8-1152k^6\lambda)]}{(\Gamma(1+\alpha))^2(1+e^{2kx})^8} + \frac{(e^{4kx}-e^{12kx})(-1024k^{11})}{(\Gamma(1+\alpha))^3(1+e^{2kx})^8} \\
& + \frac{t^{2\alpha}[(e^{6kx}+e^{10kx})(-1140k^8)]}{(\Gamma(1+\alpha))^2(1+e^{2kx})^8} + \frac{t^{3\alpha}[(e^{4kx}-e^{12kx})1536k^9\lambda]}{(\Gamma(1+\alpha))^3(1+e^{2kx})^8} \\
& + \frac{t^{2\alpha}[e^{8kx}(-5376(k^8-k^6\lambda)+k^4\lambda^2)]}{(\Gamma(1+\alpha))^2(1+e^{2kx})^8} + \frac{e^{8kx}1536k^{14}}{(\Gamma(1+\alpha))^4(1+e^{2kx})^8} \\
& + \frac{t^{2\alpha}[(e^{4kx}+e^{8kx})(2592k^6\lambda-144k^4\lambda^2)]}{(\Gamma(1+\alpha))^2(1+e^{2kx})^6} \\
& + \frac{t^{2\alpha}[(e^{2kx}+e^{14kx})(-48k^8+96k^6\lambda-36k^4\lambda^2)]}{(\Gamma(1+\alpha))^2(1+e^{2kx})^8} \\
& + \frac{t^{4\alpha}[e^{8kx}(-3456k^{10}\lambda^2-5184k^8\lambda^3)]}{(\Gamma(1+\alpha))^4(1+e^{2kx})^8} \\
& + \frac{t^{3\alpha}[(e^{6kx}-e^{10kx})(4352k^{11}-5376k^9\lambda+576k^7\lambda^2)]}{(\Gamma(1+\alpha))^3(1+e^{2kx})^8} \\
& + \frac{t^{2\alpha}[e^{6kx}(-6048k^6\lambda-432k^4\lambda^2+4224k^8)]}{(\Gamma(1+\alpha))^2(1+e^{2kx})^6}.
\end{aligned}
$$

Successively applying the algorithm given in (13) initial few terms of $u(x,t)$and$v(x,t)$ can be obtained from software package Mathematica. The approximate solution in series form is given as

$$
\begin{cases}
u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t), \\
v(x,t) = v_0(x,t) + v_1(x,t) + v_2(x,t).
\end{cases} \tag{27}
$$

Figures 7 and 9 show the surfaces of approximate solution of Eq.(2) for$u(x,t)$ which is of kink−type but bell shaped for $v(x,t)$ when $\alpha = 1$, $\beta = 1.5$, $k = 0.1$, $c_0 = 1.5$and $c_1 = 0.1$ respectively. Figures 8 and 10 show the plot of u(x,t) and v(x,t) of Eq.(2) for $k = 0.1$, $\alpha = 1.5$, $c_0 = 1.5$ and $c_1 = 0.1$, $-50 \le x \le 50$ and $t = 0.5$ respectively. It seems that all the curves of approximate solution are exactly similar with the curves of exact solutions (Fan (2001)).

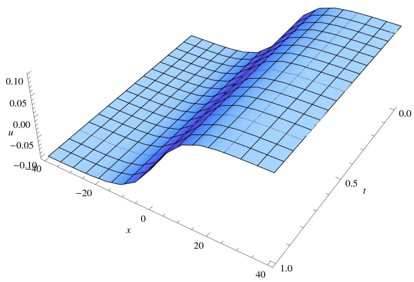

Figure 7: Approx.sol.of u(x,t) of (2) for $\alpha = 1,\ k = 0.1,\ \beta = 1.5$

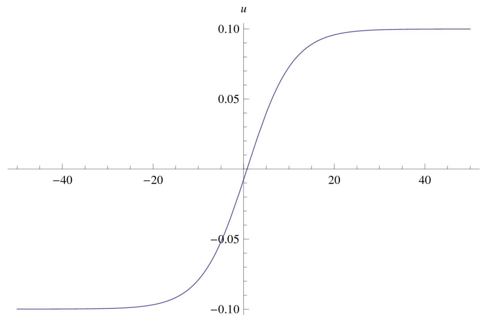

Figure 8: Plot. of u(x,t) of (2) for $\alpha = 1,\ t = 0.5$

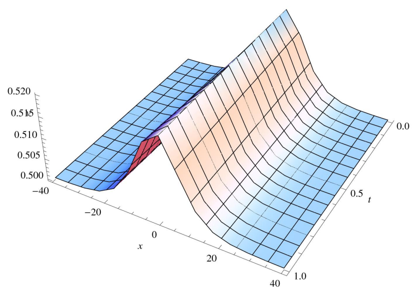

Figure 9: Approx.sol.of v(x,t) of (2) for $\alpha = 1,\ k = 0.1,\ \beta = 1.5$

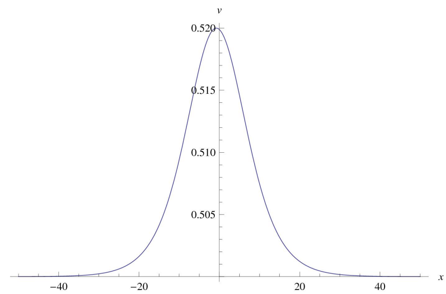

Figure 10: Plot. of v(x,t) of (2) for $\alpha = 1,\ t = 0.5$

In Tables 4 and 5, the above series solutions are tested by evaluating absolute errors between approximate solution of coupled KdV system of Equation (2) with exact solutions (Fan (2001)) given by (23). We substitute $\alpha = 1$, $\beta = 1.5$, $k = 0.1$, $c_0 = 1.5$ and $c_1 = 0.1$ in approximate solution and take only three terms of all series. Also numerical value of solution are evaluated for $\alpha = 0.5$, and 0.75. From these results it has been observed that the obtained approximate series solutions are in good agreement with the exact solutions.

Table 4: The comparison of numerical solution of u(x,t) of Eq.(2) obtained by NIM and absolute errors for difference between approximate solution for $\beta = 1.5,\ k = 0.1,\ c_0 = 1.5,\ c_1 = 0.1$ and $\alpha = 0.5,\ 0.75,\ 1$ and exact solution

			u_{NIM}		Absolute error
t	x	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 1$	$\lvert u_{exact} - u_{appx}\rvert$ for $\alpha = 1$
0.5	-50	-0.09999284	-0.09999252	-0.09999219	5.01459×10^{-9}
	-30	-0.09961012	-0.09959250	-0.09957438	2.59814×10^{-7}
	-10	-0.0808693	-0.07997782	-0.07916027	7.83142×10^{-6}
	0	-0.01204464	-0.00976739	-0.00754916	1.34731×10^{-5}
	10	0.07066489	0.07177096	0.07281809	1.61633×10^{-5}
	30	0.09937238	0.09939973	0.09942539	2.97898×10^{-7}
	50	0.09998847	0.09998897	0.09998945	5.41392×10^{-9}
1	-50	-0.09999349	-0.09999341	-0.09999325	3.86906×10^{-8}
	-30	-0.09964513	-0.09964112	-0.09963211	2.03764×10^{-6}
	-10	-0.08245136	-0.08225622	-0.08182232	1.38108×10^{-5}
	0	-0.01702887	-0.01642113	-0.01509329	1.07008×10^{-4}
	10	0.06813340	0.06845024	0.06913459	7.99211×10^{-5}
	30	0.09930891	0.09931692	0.09933415	2.43844×10^{-6}
	50	0.09998729	0.09987450	0.09998776	4.50230×10^{-8}

Table 5: The comparison of numerical solution of v(x,t) of Eq.(2) obtained by NIM and absolute errors for difference between approximate solution for $\beta = 1.5,\ k = 0.1,\ c_0 = 1.5,\ c_1 = 0.1$ and $\alpha = 0.5,\ 0.75,\ 1$ and exact solution

			v_{NIM}		Absolute error
t	x	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 1$	$\lvert v_{exact} - v_{appx}\rvert$ for $\alpha = 1$
0.5	-50	0.50000459	0.50000439	0.50000421	1.09032×10^{-6}
	-30	0.50024950	0.50023879	0.50022874	5.89524×10^{-5}
	-10	0.51001308	0.50969367	0.50938899	1.91921×10^{-3}
	0	0.51971729	0.51981413	0.51988899	2.56179×10^{-6}
	10	0.50697099	0.50722700	0.50748267	-1.91709×10^{-3}
	30	0.50015614	0.50016308	0.50017023	-5.90735×10^{-5}
	50	0.50000287	0.50000299	0.50000313	-1.09265×10^{-6}
1	-50	0.50000506	0.50000499	0.50000488	2.18992×10^{-6}
	-30	0.50027434	0.50027212	0.50026447	1.18395×10^{-4}
	-10	0.51073571	0.51064586	0.51045123	3.83649×10^{-3}
	0	0.51943449	0.51947419	0.51955588	5.05478×10^{-6}
	10	0.50643347	0.50649734	0.50663850	-3.82440×10^{-3}
	30	0.50014229	0.50014388	0.50014745	-1.18975×10^{-4}
	50	0.50000262	0.50000265	0.50000271	-2.20115×10^{-6}

5. Conclusion

In this paper, the purpose to obtain approximate solutions of time fractional generalized Hirota-Satsuma coupled KdV system and a coupled mKdV system using a coupling method which is a combination of fractional complex transform and new iterative method has been achieved. The fractional complex transform easily converts differential equations of arbitrary order into differential equations of integer order. The approximate and exact solutions are compared and observed that the obtained approximate series solutions for first few terms are very precise and converges very rapidly to the solutions of real physical problems. The results show that present technique is very reliable, accurate and most suitable for computer algorithms as well as an effective mathematical tool for many researchers working in the field of applied sciences and engineering to study linear or nonlinear fractional differential equations. Besides this, present technique can be applied using several other numerical methods as a combination with fractional complex transform to get approximate and analytical solution of fractional differential equations.

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